# EME on Millimeter Waves Part 1 - Calculations

***by Sergey Zhutyaev, RW3BP***

When we talk about EME communication on millimeter waves, we mean the 47 GHz and 76 – 80 GHz bands. Amateur EME communication at 122 GHz and above is very unlikely in the near future. The 24 GHz band also has properties similar to millimeter waves. EME communication on millimeter waves has the following features:

- The width of the main lobe an antenna suitable for EME communication is always less than the angular size of the moon. For example, my 2.4 m antenna has a main lobe width of about 0.12 degrees in the 77 GHz band. As a result of this, a calculation of EME path loss, as used on the lower bands, gives a large inaccuracy.
- Atmospheric losses, which can be neglected even in the 10 GHz band, have a significant impact.
- A very large problem is the Doppler spread of the reflected signal caused by the libration of the moon. This makes it difficult to distinguish a weak signal from the receiver noise.
- As frequency increases, it becomes more and more difficult to get the necessary power from the transmitter. On the 77 GHz band this is currently the main barrier to the first two-way EME QSO.

The list of technical difficulties can be continued, but the main problems are listed. Building an EME station requires a lot of effort, time and budget. Therefore, it is important to determine in advance what performance you want to achieve. On the lower bands the cost of errors is not so high. There will always be a number of powerful stations that will be able to have an EME QSO with you. If desired, the potential of the station can be further increased.

On millimeter wave bands that are just beginning to be used for EME communication the question is somewhat different. We need to know if it is possible to have an EME QSO with the parameters you plan on during construction of your station. What will be the signal / noise Ratio at the output of the receiver and is it possible to copy the signal at this ratio?

Now let's try to derive an expression for calculating the signal / noise ratio for EME communication on millimeter waves. First, consider the expression for EME path loss from the "ARRL Handbook" [1].

$$l = \frac{\eta\ r^2\ \lambda^2}{64\ \pi^2\ d^4} \qquad (1)$$

Where

| | | | |
|---|---|---|---|
| $r$ | is the radius of the moon | $\lambda$ | is the wavelength |
| $d$ | is the distance to the moon | $\eta$ | is the lunar reflection coefficient |

Values $r = 1.74 \times 10^6$ m, $d = 3{,}83 \times 10^8$ m, $\eta = 0{,}065$ to get the average path loss.

This expression is related to the well-known Radar equation. Indeed, EME communication is very similar to radar. For further reasoning, it is convenient to use the concept of radar cross section adopted in radar. In expression (1), the radar cross section $\sigma$ is the product of the area of the lunar disk multiplied by the reflection coefficient.

$$\sigma = \pi\, r^2\, \eta$$

Inserting σ in expression (1) and adding atmospheric losses.

$$l = \frac{\lambda^2}{64\ \pi^3\ d^4\ l_{atm}}\ \sigma \qquad (2)$$

Finally we are interested in the signal-to-noise ratio on the receiver output.

$$S/N = P_{rx} / P_{noise}$$

with

| | |
|---|---|
| $P_{rx}$ | the signal power at the receiver input; $P_{rx} = P_{tx}\, G_{rx}\, G_{tx}\, l$ |
| $P_{tx}$ | the transmitter power (W) |
| $G_{tx}$ | the transmitting antenna gain (times) |
| $G_{rx}$ | the receiving antenna gain (times) |
| $P_{noise}$ | the noise power at the receiver input; $P_{noise} = k\, T_{sys}\, B$ |
| $k = 1.38 \times 10^{-23}$ | the Boltzmann's constant (W) |
| $T_{sys}$ | the system noise temperature (K) |
| $B$ | the receiver bandwidth (Hz) |

we get the following expression:

$$S/N = P_{rx} / P_{noise} = \frac{P_{tx}\ G_{TX}\ G_{RX}\ \lambda^2}{64\ \pi^3\ d^4\ k\ T_{sys}\ l_{atm}}\ \sigma\ \frac{1}{B} \qquad (3)$$

Let us now determine how the signal/noise ratio is affected by the fact that the width of the main lobe of the antenna becomes smaller than the angular size of the moon. This means that the reflection of the signal does not involve the entire lunar disk, but only part of it, limited by the width of the main lobe. This leads to additional losses associated with the reduction of the scattering surface. For further analysis, we introduce some simplifications:

- We will assume that the lunar disk evenly reflects millimeter waves, similar to what we see with our own eyes with light.
- We will assume that the main lobe of the antenna pattern is rectangular in the Cartesian coordinate system and has a width of $\Phi_A$ equal to the half-power beam width of the real antenna $\Phi_A = \Phi_{-3dB}$.
- We introduce the parameter "m", which is equal to one if the width of the antenna beam is equal to or greater than the angular size of the moon (for small antennas). If the beam width is less than the angular size of the moon, then:

$$m = \Phi_{-3dB}\ /\ \Phi_{moon}$$

$$\Phi_{moon} = 0.5^{\circ}$$

There is a simple formula to estimate the beam width of the antenna with diameter $D_A$:

$$\Phi_{-3dB} = 70\,\lambda \,/\, D_A$$

Then

$$m = 1\,; \quad (140\,\lambda / D_A > 1) \qquad (4)$$

$$m = 140\,\lambda / D_A\,; \quad (140\,\lambda / D_A < 1) \qquad (5)$$

Based on the fact that at $m < 1$ the surface area of the moon participating in the reflection of the signal decreases proportionally to $m^2$, we obtain the following expressions for the effective scattering surface (radar cross section).

$$\sigma = \pi r^2 \eta \quad (m = 1) \qquad (6)$$

$$\sigma = \pi r^2 \eta \;(140\,\lambda / D_{AL})^2 \quad (m < 1) \qquad (7)$$

Expression (7) takes into account the fact that that if operators use different antennas, $D_{AL}$ large and $D_{AS}$ small, the size of the scattering surface is determined by a large antenna.
Let us now consider the effect of the moon's libration. To take into account the Doppler shift of the reflected signal, it is important to know at each moment the speed at which the distance from the earth observer to the moon surface is changing. At the same time the Moon is also slowly turning (libration of the moon). As a result of this rotation, an additional Doppler frequency shift of the reflected signal occurs, which is zero in the center of the disk and reaches the maximum value at the rims. The higher the moon's turning speed, the greater the additional frequency shift. If we collect the signal from the entire lunar disk, we get the frequency spread limited by the maximum values corresponding to the edges of the disk.

A formula is known [2] that allows calculation of the width of the reflected signal $S_w$ caused by the moon libration.

$$S_W = 6000\, F_{(GHz)}\, L_{R(deg/min)} \qquad (8)$$

where

$F_{(GHz)}$ is transmitter frequency,

and

$L_{R(deg/min)}$ is libration rate.
For optimal reception, the receiver bandwidth should be equal to the width of the received signal. We express the optimal bandwidth as $B_L$.

$$B = B_L$$

If we use a small antenna we collect the signal from the entire lunar disk:

$$B_L = \mathbf{S_W} \qquad (m = 1)$$

If the antenna is large, we collect the signal only from part of the moon's surface and the signal width is reduced:

$$B_L = \mathbf{S_W}\, \mathrm{m} \qquad (m < 1)$$

Prepare the result to be inserted into expression (3) for S/N and note that receiver bandwidth is included in expression as 1/B.

And note that: $F_{(GHz)} = 0.3 / \lambda_{(m)}\,; \; m = 140\,\lambda / D_A$

Then:

$$\frac{1}{B_L} = 5.6 \times 10^{-4}\, \frac{\lambda}{L_R} \qquad (m = 1) \qquad (9)$$

$$\frac{1}{B_L} = 4 \times 10^{-6}\, \frac{D_{AL}}{L_R} \qquad (m < 1) \qquad (10)$$

In the expression (10) it is taken into account that if the operators have different antennas $D_{AL}$ large and $D_{AS}$ small, the width of the reflected signal is determined by a large antenna.

Let's start with the option when both antennas are small, having the width of the beam wider than the angular size of the moon. We insert (6) and (9) to the expression (3) and note that:

$$G = \frac{4\,\pi\, S_A}{\lambda}\, \varepsilon_{ap} = \frac{\pi^2 D_A^2}{\lambda^2}\, \varepsilon_{ap}$$

Where $\varepsilon_{ap}$ the efficiency of the aperture.

We get:

$$S/N = \frac{P_{tx}\, D_{AL}^2\, D_{AS}^2}{\lambda\, T_{sys}\, L_R\, l_{atm}} \times 2 \times 10^{-5} \qquad (m = 1) \qquad (11)$$

This expression is more suitable for the lower frequency bands. On millimeter-wave bands an EME QSO with such antennas is almost impossible. It would need transmitters with an unrealistically large output power.
And finally the expression for large antennas. Insert (7) and (10) in (3). After conversion we get:

$$S/N = \frac{P_{tx}\, D_{AL}\, D_{AS}^2}{T_{sys}\, L_R\, l_{atm}} \times 2.8 \times 10^{-3}\,; \; if \;(m < 1) \qquad (12)$$

This is the desired formula for determining the possibility of EME QSO on millimeter waves. It shows the signal-to-noise ratio at the receiver output having a bandwidth matched with the width of the signal reflected from the moon. In this case, the boundary signal-to-noise ratio to copy the signal corresponds to the level of S/N = 1 or (0 dB). At lower values of S/N requires additional filtering by averaging repeated many times on the CW or the digital signal. It is important to remember that *Tsys* in this formula is the total noise temperature of the entire receiving system, including receiver noise, antenna noise, moon noise and atmosphere noise.

Now let's try to compare the results of the calculation by the expression (12) with the practical results obtained on 77 GHz band in February 2013 [3]. The best results were obtained on 25 February under the following conditions. Air temperature -1°C, humidity 70%, the elevation of the moon 35 degrees.
The calculator "ATMOSPHERE (VK3UM)" [4] shows a total atmospheric loss of 2 dB (1.6 times) for these conditions.

Calculator "LIBCALC (VK3UM)" [5] for the moment of the echo test shows the value of the libration rate 0.002 degrees per minute.
The transmitter power was 60 watts. Noise temperature of the receiving system was 1200K. The antenna diameter was 2.4 meters.
If we insert these parameters in expression (12) we get:

$$S/N = 0{,}6\ (-2.2\ dB)$$

To calculate the receiver bandwidth, which corresponds to this result, let us use the expression (10):

$$B_L = 210\ Hz$$

If we recalculate this result with a more familiar bandwidth of 2500 Hz, we get:

$$S/N = -13\ dB;\quad B = 2500\ Hz$$

Analysis of the audio files recorded on February 25, 2013 shows the signal / noise ratio of approximately -12 dB in the 2500 Hz band, which is very close to the calculated value. Despite some simplifications in the derivation of the expression (12) a good agreement between the calculation and the experiment is obtained.

### What is the benefit of the expression (12)?

- The expression allows calculation of the possibility of EME communication on millimeter waves with the available parameters of the station.
- The expression allows calculation of the possibility of EME communication on new millimeter wave bands. The next higher amateur bands are at frequencies above 120 GHz. There is no frequency or wavelength in expression (12). This means that the parameters of the station allowing EME communication on 47 GHz or 77 GHz bands should be kept at frequencies above 120 GHz. However, it is extremely difficult to obtain several tens of watts at these frequencies and maintain the efficiency of a 2.4 m antenna.

- The expression allows you to calculate the expected signal / noise ratio in the case of two antennas of different sizes. For the first unsuccessful attempt to copy my signal on 77 GHz band by Al W5LUA, I used a 1 m dish. Only after changing to the 2.4 m antenna was the signal detectable. The calculation shows that the reduction the size of the antenna to 1 m leads to a degradation in the signal-to-noise ratio of 5.8 times or 7.6 dB. That's too much.
- It is seen that in the absence of a large reserve of system capability it is very important to choose the optimal time for communication. It is important to choose the time corresponding to the minimum value of the libration rate. To reduce atmospheric losses, minimum air humidity and the maximum possible antenna elevation are desirable. If the Moon is low above the horizon, the loss in the atmosphere increases dramatically. This significantly complicates the communication across large distances. Thus, to the classical requirement of the minimum earth-Moon distance on millimeter waves, a number of requirements are added, which often are in mutual contradiction. We have to look for a compromise.

### References

[1] Earth-Moon-Earth (EME) Communication (from ARRL Handbook, 2010):
http://physics.princeton.edu/pulsar/K1JT/Hbk_2010_Ch30_EME.pdf

[2] "Predicting Libration Fading on the EME Path", 14th International EME Conference, Dallas, August 2010 and DUBUS 3/2010:
http://www.vk3um.com/G3WDG_libration%20paper%20revised.pdf

[3] RW3BP EME test on 77.5 GHz:
https://www.youtube.com/watch?v=2En_W2EaJFw

[4] Atmosphere Loss Calculator by VK3UM:
http://www.vk3um.com/atmosphere%20calculator.html

[5] Libration Calculator by VK3UM:
http://www.vk3um.com/libration%20calculator.html